BSc [Mathematics - I], SEM - I, Ist YEAR.

UNIT - I

1. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $(x \neq y)$. S.T.
(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$
(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u$

2. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then S.T. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan^3 u$

3. If $u = \log(\tan x + \tan y + \tan z)$ then evaluate
$(\sin 2x)\frac{\partial u}{\partial x} + (\sin 2y)\frac{\partial u}{\partial y} + (\sin 2z)\frac{\partial u}{\partial z}$

4. If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2 \neq 0$ S.T. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

5. If $u = \frac{y^3-x^3}{y^2+x^2}$ S.T. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$

6. Discuss continuity of $f(x,y) = \begin{cases} \frac{x^2}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ at origin

7. Discuss the Continuity of
$f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ at origin.

8. If $z(x,y) = (x+3y)^{3/2} + (x-4y)^{7/2}$ then evaluate
$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 12\frac{\partial^2 z}{\partial y^2} = 0$

9. If $u = \log(x^3+y^3+z^3-3xyz)$ then S.T
(i) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$
(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{-3}{(x+y+z)^2}$

(10) State and prove Euler's theorem on homogeneous function of two Variables.

(11) Evaluate $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$.

(12) If i) $f(x,y) = \frac{xy}{x^2+y^2}$ ii) $f(x,y) = (x^2+y^2)e^{x-y}$ then find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y^2}$ (or) f_{xx}, f_{xy}, f_{yy}

(13) If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, $xy \neq 0$ p.T. $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$

(14) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then s.T. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

(15) If $z = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$. then s.T. $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\cot z$.

(16) If $u(x,y) = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(17) If $u = f(r)$, $r = \sqrt{x^2+y^2}$ p.T. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$

(18) If $u = e^x(x\cos y - y\sin y)$, s.T. $u_{xx} + u_{yy} = 0$.

UNIT - II

1. Find the minimum value of $x^2+y^2+z^2$ given that $ax+by+cz=p$.
2. Find the maximum value of $x^2+y^2+z^2$ given that $xyz=a^2$.
3. Find the extreme values of $f(x,y,z)=xyz$ when $x+y+z=12$
4. S.T. the minimum value of $u=xy+\frac{a^3}{x}+\frac{a^3}{y}$ is $3a^2$
5. If $f(x,y)$ possesses continuous second order partial derivatives f_{xy} and f_{yx} then S.T. f_{xy} and f_{yx}
6. If $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ then S.T. $f_{xy}(0,0)\neq f_{yx}(0,0)$.
7. Discuss the maximum or minimum value of $u=x^3y^2(1-x-y)$
8. State and prove Taylor's theorem for function of two variables.
9. If $H=f(y-z,z-x,x-y)$, P.T. $\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}=0$
10. If $z=x^2+y^2$, $x=at^2$, $y^2=2at$ then evaluate $\frac{dz}{dt}$
11. Find $\frac{du}{dt}$, when $u=\sin\left(\frac{x}{y}\right)$, $x=e^t$, $y=t^2$
12. Find the derivative of i) $f(x,y)=x^3+y^3-3ax^2=0$ ii) $f(x,y)=(\cos x)^y-(\sin y)^x=0$ (iii) $x^y=y^x$.
13. Find the total derivative of $u(x,y,z)=e^{xyz}$
14. If $f(x,y)=\cos^{-1}\left(\frac{y}{x}\right)$ then find total differential of f

(15) If $z = z(x,y)$ and $x = e^{2u} + e^{-2v}$, $y = e^{2u} - e^{-2v}$, then evaluate $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$.

(16) using Taylor's theorem Expand

(i) $f(x,y) = x^2 + xy + y^2$ in powers of $(x-1)$ & $(y-2)$

(ii) $f(x,y) = e^y \log_e(1+x)$ in powers of x & y at $(0,0)$

(iii) $f(x,y) = x^2 + xy - y^2$ in powers of $(x-1)$ & $(y+2)$

(iv) $f(x,y) = e^{x+y}$ at $(0,0)$ for $n=3$

(17) i) $(\tan x)^y + y^{\cot x} = a$ then find $\frac{dy}{dx}$.

ii) If $x^3 + y^3 = 3axy$ then find $\frac{d^2y}{dx^2}$.

(18) Prove that if $y^3 - 3ax^2 + x^3 = 0$ then $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$

(19) If $u = \log(x+y+z)$, $x = e^{-t}$, $y = \sin t$, $z = \cos t$, then evaluate $\frac{du}{dt}$.

(20) If $z = e^u f(v)$, $u = ax+by$, $v = ax-by$, then S.T.

$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

UNIT - III

1. Find the evolute of the Curve i) $x^2 = 4ay$ (ii) $y^2 = 4ax$.
2. Find the envelope of the family of Straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a an b are Connected by the relation $a^2 + b^2 = c^2$, c is Constant.
3. Find the evolute of the Curve
   (i) $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$
   (ii) $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, $y = a\sin t$
   (iii) $x = a\cos^3\theta$, $y = a\sin^3\theta$
4. Find the envelopes of the Curves
   (i) $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1$ where $a^n + b^n = c^n$.
   (ii) $x\cos\alpha + y\sin\alpha = l\sin\alpha\cos\alpha$
   (iii) $y^2 = \alpha^2(x - \alpha)$, α is parameter
   (iv) $y = mx + \frac{a}{m}$, m is parameter
   (v) $y = mx + am^3$, m is parameter
   (vi) $y = mx + \sqrt{1+m^2}$, m is parameter.
   (v) $y = mx + 2m^3$, m is parameter
5. Find the envelope of the family of Straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a & b are connected by relation $ab = c^2$, c is Constant.
6. Find the circle of Curvature at the Curve $y^2 = 4ax$ at $P(am^2, 2am)$
7. Find the radius of Curvature of the Curve
   (i) $x = y^2 + 4y + 03$ at $P(8,1)$
   (ii) $x^3 + xy^2 - 6y^2 = 0$ at $(3,3)$
   (iii) $y = 30/x$ at $P(3,10)$
   (iv) $y = c\cosh\frac{x}{c}$ at any point
   (vi) $y^2(a-x) = x^2(a+x)$ at origin

(8) Find the Centre of Curvature of the Curve
(i) $x^3 + y^3 = 2$ at $(1,1)$
(ii) $xy(x+y) = 2$ at $(1,1)$

(9) Find the chord of Curvature through the pole for the cardioid $r = a(1+\cos\theta)$

(10) S.T. the Curvature of point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the folium $x^3 + y^3 = 3axy$ is $-8\sqrt{2}/3a$. Define Curvature of Curve.

(11) S.T. the radius of Curvature of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at an end of major axis is equal to Semi-latus rectum of Ellipse.

(12) S.T. the evolute of ellipse $x = a\cos\theta$, $y = b\sin\theta$ is $(ax)^{2/3} + (by)^{2/3} = (a^2-b^2)^{2/3}$

(13) Using Newton's method, find radius of curvature for the Curve $x^3 + y^3 - 2x^2 + 6y = 0$ at origin $O(0,0)$.

Teacher's Signature :

UNIT – IV

① Find the length of the curve

(i) $y = \log_e\left(\frac{e^x - 1}{e^x + 1}\right)$ from $x = 1$ to $x = 2$

(ii) $9y^2 = (x-2)(x-5)^2$ (vii) $y = \log \tan\left(\frac{x}{2}\right)$ from $x = 1$ to 2

(iii) $y = x^{3/2}$ from $x = 0$ to $x = 4$

(iv) $x = e^\theta \sin\theta$, $y = e^\theta \cos\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$

(v) $y = x\sqrt{x}$ from $x = 0$ to $x = 4/3$

(vi) $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $0 \le t \le \frac{\pi}{4}$

② Find the length of the Asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ where a is constant

③ S.T. length of the curve $x^2 = a^2(1 - e^{x/a})$ measured from $O(0,0)$ to $P(x, y)$ is $a\log\left(\frac{a+x}{a-x}\right) - x$.

④ Find the volume of solid of revolution generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major axis (or) X-axis and minor axis (or) y-axis.

⑤ Find the volume of solid generated by revolving one arc of cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about X-axis.

⑥ Find the volume of solid generated by revolving the cardioid $r = a(1 + \cos\theta)$ about initial line.

⑦ Find the surface area of solid generated by revolving

(i) $x^2 + 4y^2 = 16$ about its major axis.

(ii) $y = c\cosh\left(\frac{x}{c}\right)$ about X-axis

(iii) $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about line $y = 0$

(iv) $r = a(1 - \cos\theta)$ about initial line.

⑧ State and prove Pappus theorem for volume of solid of revolution.

Expt. No. :

Page No :

9. Find the Surface area of a Sphere of radius 'a'.
10. Find the Volume of Solid generated by revolving
(i) $y = x^2$, $x = 3$ and x-axis
(ii) $y = x^3$, $y = 0$, $x = 2$ about x-axis
(iii) $y^2 = 12x$ about x-axis from $x = 0$ to $x = 5$
(iv) $y = \cos x$, $y = 0$ from $x = 0$ to $\pi/2$ about x-axis.
11. State and prove pappus theorem for Surface of revolution.
12. State and prove volume of revolution
13. State and prove Surface of revolution.

Teacher's Signature :