Code No: E-10005/BL

FACULTY OF SCIENCE
B.A. / B.Sc. (CBCS) I Semester (Backlog) Examination, June / July 2023
Subject: Mathematics
Paper – I: Differential and Integral Calculus

Time: 3 Hours

Max. Marks: 80

PART – A

(8 x 4 = 32 Marks)

Note: Answer any eight questions.

1. If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ show that $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$.

2. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

3. Verify that if $z = xy\, f(y/x)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$.

4. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2y,\ x = at^2, y = 2at$.

5. If $F(x, y, z) = 0$ find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

6. State Taylor's theorem for function of two variables.

7. Define radius of curvature.

8. Find the envelope of the straight lines $x\cos\alpha + y\sin\alpha = l\sin\alpha\cos\alpha$, α is parameter.

9. Find $\frac{ds}{dt}$ for the curve $x = a(t - \sin t), y = a(1 - \cos t)$.

10. Find the perimeter of the circle $x^2 + y^2 = a^2$.

11. Find the length of the arc of the curve $y = \log\sec x$ from $x = 0$ to $x = \pi/4$.

12. Find the volume of the hemisphere.

PART – B

(4 x 12 = 48 Marks)

Note: Answer all the questions.

13. a) State and prove Euler's theorem on homogeneous functions.

(OR)

b) If $u = \tan^{-1}\left(\frac{x+Y}{\sqrt{x}+\sqrt{y}}\right)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{4}\sin 2u$.

14. a) Expand $\sin xy$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{2}\right)$ upto second degree terms.

(OR)

b) Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$ for the function f given by

$f(x, y) = \frac{xy(x^2-y^2)}{x^2+y^2}; (xy) \neq (0,0),\ f(0,0) = 0$.

15. a) Show that the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$.

(OR)

b) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ when $ab = c^2$, c is constant.

16. a) Find the volume of the right circular cone of height h and base of radius a.

(OR)

b) Find the length of the curve $y = \log\frac{e^1-1}{e^x+1}$ from $x = 1$ to $x = 2$.

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