SEMESTER – I

MATHEMATICS – I [DIFFERENTIAL AND INTEGRAL CALCULUS]

IMP QUESTIONS

UNIT – I

SHORT ANSWERS

1) Explain function of Several variables

2) If $z = f(x+ay) + \phi(x-ay)$, Then P.T. $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$

3) If $u = e^{xyz}$ then S.T. $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2) e^{xyz}$

4) If $u = (x^2+y^2+z^2)^{1/2}$ then S.T. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{u}$.

5) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2+y^2+z^2 \neq 0$, then S.T. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

6) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ then Evaluate $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

7) By Using Euler's Theorem S.T. $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{4} \sin 2u = 0$ where $u = \cot^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$.

8) If $u = f\left(\frac{y}{x}\right)$ then S.T. $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$.

9) If $u = \log(x^3+y^3+z^3-3xyz)$, then S.T. $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

10) If $u = x^2y + y^2z + z^2x$ then Evaluate $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

UNIT – I

LONG ANSWERS

1) State and prove Euler's theorem for Homogeneous functions.

2) If $u = \log(x^3+y^3+z^3-3xyz)$ then S.T. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

3) If $u = \log\left[\frac{x^4+y^4}{x+y}\right]$ then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

4) If $z = f(x,y)$ is a Homogeneous fn of x, y of degree n. then S.T. $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$

5) If $u = \cos^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ then P.T. $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \cot u$.

6) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then S.T. $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

7) If $a^2x^2 + b^2y^2 - c^2z^2 = 0$ then S.T. $\frac{1}{a^2} \frac{\partial^2 z}{\partial x^2} + \frac{1}{b^2} \frac{\partial^2 z}{\partial y^2} = \frac{1}{c^2 z}$

8) If $\sin v = \frac{(x+2y+3z)}{\sqrt{(x^8+y^8+z^8)}}$ then S.T. by Euler's Theorem $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + z \frac{\partial v}{\partial z} + 3 \tan v = 0$.

9) If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ then P.T. $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$.

10) If $u = \log(\tan x + \tan y + \tan z)$ then P.T. $(\sin 2x) \frac{\partial u}{\partial x} + (\sin 2y) \frac{\partial u}{\partial y} + (\sin 2z) \frac{\partial u}{\partial z} = 2$.

UNIT – II

SHORT ANSWERS

1) If $u = \log(x+y+z)$, $x = \cos t$, $y = \sin^2 t$, $z = t^2$ then find $\frac{du}{dt}$.
2) If $w = x^2+y^2$, $x = r-s$ and $y = r+s$ then evaluate $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$
3) Find $\frac{dz}{dt}$ when $z = xy(x+y)$, $x = at^2$, $y = 2at$
4) If $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$, $y = t^2$, then evaluate $\left(\frac{du}{dt}\right)$
5) If $z = \log(u^2+v)$, $u = e^{x+y^2}$, $v = x+y^2$ then find $2y\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$
6) Define Total Differentiation

Write about
   a) Derivatives of composite fn.
   b) Derivatives of Implicit fn.

7) If $x = e^u + e^{-v}$, $y = e^{-u} - e^{-v}$ then S.T. $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.
8) If $H = f(y-z, z-x, x-y)$ then P.T. $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$.
9) If $x^y = y^x$ then find $\frac{dy}{dx}$.
10) If $x^y + y^x = a^b$, then find $\left(\frac{dy}{dx}\right)$.

LONG ANSWERS

1) If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$, then S.T. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
2) Find $\frac{d^2y}{dx^2}$ If $x^3+y^3 = 3axy$.
3) State and prove Taylor's theorem
4) Expand $f(x,y) = x^2y + 3y - 2$ in terms of $x+1, y-2$ by Taylor's series.
5) Expand $f(x,y) = x^2 + xy - y^2$ by Taylor's theorem in terms of $(x-1)$ and $(y+2)$
6) Discuss the Max (or) Min values of $u = x^3+y^3-3axy$
7) Discuss the Max and Min values of $f(x,y) = xy + \frac{9}{x} + \frac{3}{y}$.
8) Discuss the Max and Min values of $f(x,y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$
9) Find the Maximum value of $x+y+z$, subject to the condition $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$.
10) Find minimum value of $x^2+y^2+z^2$ when $x+y+z = 3a$.

UNIT-III

SHORT ANSWERS

1) Define (a) Curvature
(b) Radius of Curvature
(c) Evolute (d) Envelope.

2) Find the Envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = a$ where α is a parameter

3) Find Envelope of the Curve $my + m^2 x - 10 = 0$, where 'm' is a parameter

4) Find Envelope of family of straight lines $y = mx + \sqrt{a^2m^2+b^2}$.

5) Find $\frac{ds}{dx}$ for the curve $a\log\left[\frac{a^2}{a^2-x^2}\right]$

6) Find Co-ordinates of the Centre of Curvature of the parabola $y^2 = 4ax$.

7) Find the Radius of Curvature of $\sqrt{x}+\sqrt{y}=\sqrt{a}$, at the point where the line $y = x$ cuts it.

x

LONG ANSWERS

1) P.T. the Radius of Curvature at the point $(-2a, 2a)$ on the curve $x^2y = a(x^2+y^2)$ is $-2a$.

2) P.T. the Evolute of the parabola $y^2=4ax$ is $27ay^2 = 4(x-2a)^3$.

3) Find the Evolute of the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

4) Find the Envelope of the straight line $x\cos t + y\sin t = a + a\cos t \log\tan\frac{t}{2}$ where t is a parameter.

5) Find the Radius of Curvature at the origin for the curve $x^4 - y^4 + x^3 - y^3 + x^2 - y^2 + y = 0$.

x

UNIT-IV

SHORT ANSWERS

1) Find the Length of Arc of the curve $r = ae^{\theta\cot\alpha}$ taking $s = 0$ when $\theta = 0$

2) Find the length of Arc of the catenary $y = c\cosh\left(\frac{x}{c}\right)$ Measured from the vertex $(0, c)$ to any point (x, y)

3) Find the volume generated by the revolution of an arc of the catenary $y = c\cosh\frac{x}{c}$, about the axis of x between $x = a$ & $x = b$.

4) Find the surface generated by the revolution of an arc of the catenary $y = c\cosh\frac{x}{c}$ about the axis of x.

5) State and prove Pappus theorem for volumes.

x

LONG ANSWERS

1) Find the whole length of the Asteroid $x^{2/3}+y^{2/3}=a^{2/3}$

2) P.T. the whole length of the curve $x^2(a^2-x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$.

3) Find the length of the arc of the parabola $y^2=4ax$ cut off by its latus rectum.

4) Find the perimeter of the cardioid $r = a(1-\cos\theta)$.

5) Find the volume of the solid obtained by revolving the Ellipse $\frac{x^2}{a^2} = \frac{y^2}{b^2} = 1$ about the axis of x.

6) Find the surface of the solid obtained by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line

7) Find the surface of the solid generated by revolution of the curve $x^2+4y^2=16$ about the x-axis.

8) S.T. the volume of the solid generated by the revolution of the cycloid $x = a(\theta+\sin\theta)$, $y = a(1-\cos\theta)$ about the y-axis is $\pi a^3\left(\frac{3}{2}\pi^2 - \frac{8}{3}\right)$

x