Code No: F-15005

FACULTY OF SCIENCE

B.A./B.Sc. (CBCS) I-Semester Examination, December 2023/January 2024

Subject: Mathematics

Paper – I : Differential and Integral Calculus

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8x4= 32 Marks)**

1. Evaluate $\lim_{\substack{x\to\infty \\ y\to 2}} \frac{xy}{x^2+2y^2}$

2. If $u = e^{x^y}$ then find $\frac{\partial^2 u}{\partial y \partial x}$

3. If $u = e^{ax} sinby$ then find $\frac{\partial^2 u}{\partial x \partial y}$ and $\frac{\partial^2 u}{\partial y \partial x}$

4. If $z = e^{xy}$ where $x = t\, cost, y = t\, sint$ then find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$

5. Find $\frac{dy}{dx}$ if $(x)^y = (y)^x$

6. Find the stationary points of the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$

7. Find the radius of curvature at each point $P(\varphi, s)$ of the curve $s = c\, log(sec\varphi)$

8. Find the radius of curvature at the origin for the curve $x^3 + 3x^2y - 4y^3 + y^2 - 6x = 0$

9. Find the envelope of the families of the curve $\frac{x\, cos\alpha}{a} + \frac{y\, sin\alpha}{b} = 1$; where α is a parameter.

10. Find the length of arc of the catenary $y = c\, cosh\left(\frac{x}{c}\right)$ measured from the vertex to the point $P(x, y)$.

11. Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the $x-$axis.

12. Find the surface area of the sphere of radius a

PART – B

Note: Answer all questions. **(4 x 12 = 48 Marks)**

13. (a)(i) If $u = x^y$ then show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$

(ii) If $z = \tan(y + ax) + (y - ax)^{3/2}$ then find the value of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$

(OR)

(b)(i) If $u = \log(tanx + tany + tanz)$ then find the value of $sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z}$

(ii) Verify Euler's theorem for $z = sin^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{y}{x}\right)$

14. (a)(i) If $z = f(x, y)$; $x = e^{u} + e^{-v}$ and $y = e^{-u} - e^{v}$ then show that
$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$

(ii) If $u = x^2+y^2 + z^2$, where $x = e^t$, $y = e^t sint$ and $z = e^t cost$ then find $\frac{du}{dt}$

(OR)

(b) (i) Discuss the maxima or minima of the function $f(x, y) = 3x^2 - y^2 + x^2$

(ii) Expand the function $f(x, y) = e^x \log(1 + y)$ in Taylor's expansion about $(0,0)$ up to second degree term.

15. (a) (i) For the curve $y = \frac{ax}{a+x}$, if $'\rho'$ is the radius of curvature of at any point $P(x, y)$ then show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2$

(ii) Show that there is no envelope for the family of circles with centres $(\alpha, 0)$ and radii α^2 where α is a parameter.

(OR)

(b) (i) Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$

(ii) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are parameters connected by $a + b = c$

16. (a) (i) Find the length of the complete arch of the cycloid $x = a(\theta - sin\theta), y = a(1 - cos\theta)$.

(ii) Find the area of the surface of revolution generated by revolving about the $y -$ axis the arc of $x = y^3$ from $y = 0$ to $y = 2$

(OR)

(b) (i) Find the perimeter of the cardioid $r = a(1 + cos\theta)$.

(ii) Find the volume of a spherical cap of height h cut-off from a sphere of radius $'a'$

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